

Information free quantum bus for generating stabiliser states

Simon J. Devitt, Andrew D. Greentree, Lloyd C.L. Hollenberg¹

¹ *Centre for Quantum Computer Technology, School of Physics
University of Melbourne, Victoria 3010, Australia.*

(Dated: February 1, 2008)

Efficient generation of spatially delocalised entangled states is at the heart of quantum information science. Generally flying qubits are proposed for long range entangling interactions, however here we introduce a bus-mediated alternative for this task. Our scheme permits efficient and flexible generation of deterministic two-qubit operator measurements and has links to the important concepts of mode-entanglement and repeat-until-success protocols. Importantly, unlike flying qubit protocols, our bus particle never contains information about the individual quantum states of the particles, hence is information-free.

PACS numbers: 03.67.Hk, 03.67.Mn, 03.67.Lx

INTRODUCTION

Recent research into quantum information and computation has not only spawned a number of architecture proposals for quantum computation (QC), but also proposals for useful technologies based on smaller, entangled quantum systems. Quantum key distribution [1], quantum dense coding [2] and the improvement of frequency standards using entangled systems [3] are strong examples of how highly entangled states can be exploited to build novel devices: The incorporation of such entangled-state protocols with existing micro- and nano-scale fabrication is an enormous opportunity for the semiconductor industry.

The development of viable quantum computers and preparation of effective multi-qubit entangled states depends crucially on transport protocols that can be used to shuttle quantum information and to allow for interactions between isolated qubits. Effective transport of quantum information is essential to the scaling of small, functional elements, and will enable an interpolation between small scale devices and full-blown, massively entangled quantum computers. In the solid-state, inevitable fabrication errors also enforce the need for defect-tolerant methodologies, and transport allows for natural mechanisms to incorporate such features.

Ion traps [4, 5] and photonic based quantum computers [6] generally allow for easy and quick long range qubit transport. On the other hand, solid-state architectures [7, 8, 9, 10, 11, 12, 13] are often limited to nearest neighbour interactions on a linear array of qubits, leading to several problems: For example, qubit transport in linear systems is generally proposed using SWAP operations which reduce the threshold for concatenated error correction and unless modifications to the underlying architecture are made, such schemes are not Fault-Tolerant [14].

The concept of flying qubits and quantum bus systems has received significant attention [15, 16, 17, 18, 19] as a means to combat the problem of long range transport in systems that do not exhibit them naturally. Bose introduced quantum state transfer via unmodulated spin chains [20], while teleportation hubs [21] have been proposed to combat long range transport in linear systems. However, these trans-

port hubs do not remove the need for SWAP gates between qubits *not* adjacent to hubs and generally do not allow small multi-qubit state preparation without considerable resource overhead. Here, we show a transport protocol that mediates entangling operations between isolated data qubits, without communicating single-qubit states directly. We show that this scheme is extremely flexible and efficient in generating multi-qubit entangled states and operates in a fundamentally different fashion from conventional gate-driven entangling operations. Furthermore, the flexibility of our scheme relaxes some of the requirements for controllability, and hence could be an enabler for new approaches to quantum computing.

Along with flying qubit schemes and quantum bus systems, interactions on well isolated data qubits can be achieved using measurement based quantum computation. The two main concepts are teleportation based QC (TQC) [22, 23, 24, 25] and cluster state QC (CSQC) [26, 27]. These ideas differ significantly from the traditional circuit based paradigm in that interactions are not performed using unitary gates. TQC uses correlated multi-qubit measurements and appropriate ancilla states to perform operations via teleportation. CSQC requires an initial, highly entangled, multi-qubit cluster (or general graph state [28]) and arbitrary single qubit measurements.

To mediate interactions between two qubits, we introduce a bus qutrit. As an example we show a spatial qutrit defined over three sites, which we label Alice, Bob₁, and Bob₂, $|A\rangle$, $|B_1\rangle$ and $|B_2\rangle$ respectively. The qutrit takes on the role of the ancilla used in standard two-qubit operator measurements [29]. Our scheme utilises several properties from circuit, teleportation and cluster state computation, presenting a hybrid protocol that could be used to achieve universality.

QUTRIT TRANSFER PROTOCOL

To generate operator measurements, the qutrit is placed into a superposition of well-defined non-local spatial states, adjacent to separate isolated data qubits. The spatial state of the qutrit is then used as a control for unitary gates on the data qubits. This distinguishes our protocol from traditional fly-

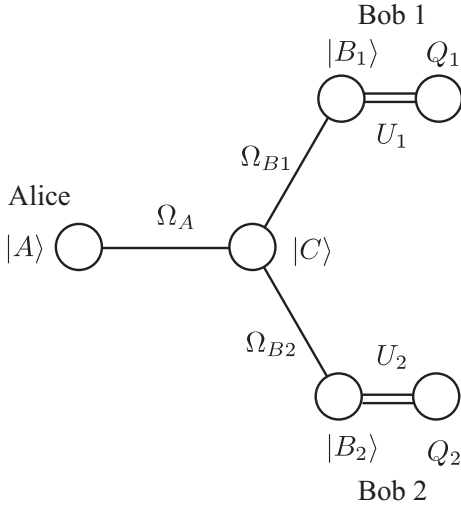


FIG. 1: Schematic of the configuration required to demonstrate the qutrit transport protocol using a spatially defined bus qubit. In common with communications approaches, we define the starting site as Alice, and the two recipient points are Bob₁ and Bob₂. Using the multi-recipient adiabatic passage (MRAP) protocol, the central site is never occupied. By using counter-intuitive pulse ordering and varying the relative intensities of the tunnelling matrix elements, it is possible for Alice to send the bus particle to an arbitrarily weighted superposition of the Bobs, although for our purposes the equally weighted superposition is chosen. Here the single lines correspond to controlled tunnelling matrix elements for the single control particle Hamiltonian, and the double lines correspond to the controlled interactions between the Bob sites and the qubits. The parity result from the two-qubit operator measurement on Q_1 and Q_2 is effected by post-selecting the state of the qutrit following the reversal of the protocol.

ing qubit schemes in that computational information stored on each qubit is never transferred to or by the bus, justifying the term *information-free*. Some potential implementations include an electron that can be placed into a physically delocalized superposition around data qubits, or a photon pulse that can be placed into a superposition of spacio-temporal modes.

For clarity, we will concentrate on spatially delocalised electrons, using the specific protocol of multi-recipient adiabatic passage (MRAP) [30]. MRAP is a protocol for adiabatically generating spatially delocalised superposition states appropriate for solid-state quantum devices. This case is illustrated in Fig. 1, and we follow the sender/receiver notation commonly used in communications theory and in Ref. [30]. Note that the counter-intuitive pulse sequence used for MRAP excludes population from the central dot (labeled $|C\rangle$) and hence we ignore this state in our analyses that follow.

We present the Qutrit Transfer Protocol (QTP) as a standard transformation that can be applied to a given system, demonstrating the equivalence of this scheme to measurement of the operators $XX = \sigma_x \otimes \sigma_x$ and $ZZ = \sigma_z \otimes \sigma_z$. Using the QTP and the ability to perform single qubit operations directly on data qubits we demonstrate linear cluster state preparation and universal computation.

The general transformations for a qutrit defined by a source state, (Alice, $|A\rangle$) and target states (Bobs, $|B_1\rangle$, $|B_2\rangle$) is described by the vector, $(|A\rangle, |B_1\rangle, |B_2\rangle)^T$. The essential transformation matrix can be written as,

$$U_{\text{QTP}} = \begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/2 & -1/2 \\ 1/\sqrt{2} & -1/2 & 1/2 \end{pmatrix}. \quad (1)$$

As stated above, the MRAP protocol provides a natural method for generating the required qutrit transformations, and we briefly review these for clarity. The form of a four-site structure as shown in Fig. 1 is isomorphic to the well-known tripod atom familiar to quantum optics. In particular, we can apply techniques for generating arbitrary ground-state superpositions that have already been developed in the optical regime in this case [31, 32]. Defining the coupling between each site and the central dot as Ω_α for $\alpha = A, B_1, B_2$ (assumed real and positive), where Alice and each Bob has control of the energy of the bus-particle on their site, which has been assumed equal, and their appropriate tunnel matrix element, we have the Hamiltonian

$$\mathcal{H} = \Omega_A(t)|C\rangle\langle A| + \Omega_{B_1}(t)|C\rangle\langle B_1| + \Omega_{B_2}(t)|C\rangle\langle B_2| + \text{h.c.} \quad (2)$$

where the time-varying tunneling matrix elements are controlled, for example, by local control of surface gate potentials.

The relevant states for MRAP state transfer are those with zero energy eigenvalue, which are given by the null-space of \mathcal{H} . These are

$$|D_1\rangle = \frac{\Omega_{B_1}}{\sqrt{\Omega_A^2 + \Omega_{B_1}^2}}|A\rangle - \frac{\Omega_A}{\sqrt{\Omega_A^2 + \Omega_{B_1}^2}}|B_1\rangle, \quad (3)$$

$$|D_2\rangle = \frac{\Omega_{B_2}}{\sqrt{\Omega_A^2 + \Omega_{B_2}^2}}|A\rangle - \frac{\Omega_A}{\sqrt{\Omega_A^2 + \Omega_{B_2}^2}}|B_2\rangle, \quad (4)$$

where we have dropped the time dependence of the Ω . Of course, any superposition of these vectors is also in the null space, so if the system remains in the null-space, $|C\rangle$ will never be populated, hence the definition of the bus particle in this case as a qutrit defined over sites $|A\rangle$, $|B_1\rangle$, and $|B_2\rangle$. In particular, we set $\Omega_{B_1} = \Omega_{B_2} = \Omega_B = \Omega_B^{\text{max}}[1 - \text{erf}(t/\sigma)]/2$ and $\Omega_A = \Omega_A^{\text{max}}[1 + \text{erf}(t/\sigma)]/2$, where σ is the width of the roll off of the error function (erf), Ω_A^{max} and Ω_B^{max} are the maximum values of the tunneling matrix elements. Without presenting details here, we simply note that either σ or the Ω^{max} should be chosen to maintain adiabaticity, then the state that adiabatically connects Alice to each of the Bobs is

$$|D_3\rangle = \frac{2\Omega_B|A\rangle - \Omega_A(|B_1\rangle + |B_2\rangle)}{\sqrt{4\Omega_B^2 + 2\Omega_A^2}}. \quad (5)$$

TWO-QUBIT OPERATOR MEASUREMENTS

Consider two data qubits $[Q_1, Q_2]$ that can be coupled to the qutrit through a controlled two-particle entangling operation.

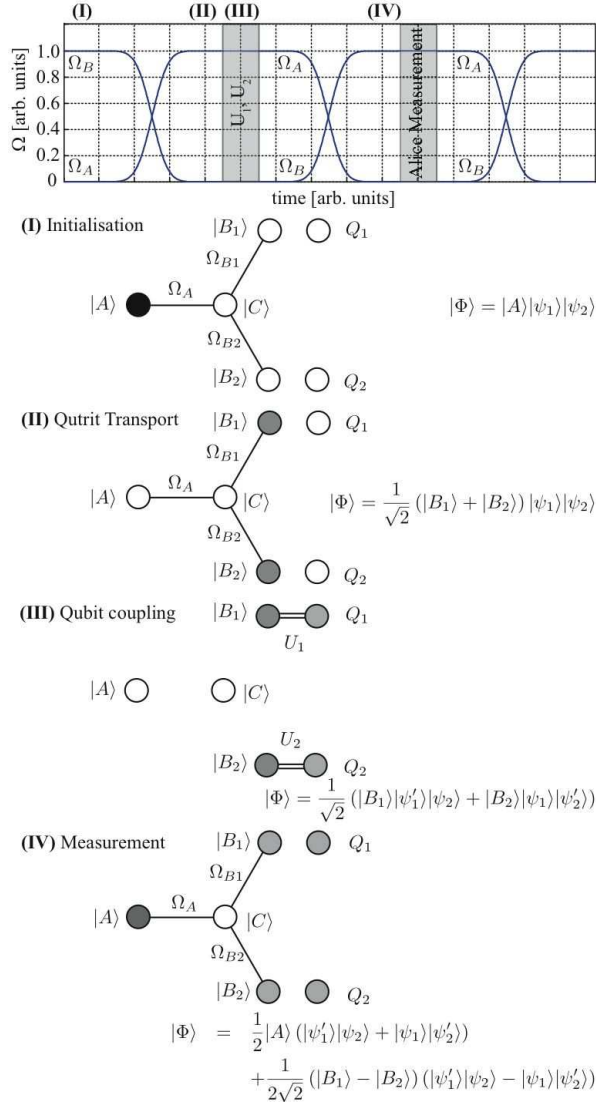


FIG. 2: Pulsing sequence for tunneling matrix elements (top) and schematics showing system evolution through the MRAP protocol to realise qutrit transport and two-qubit operator measurements. (I) The qutrit is initialised at $|A\rangle$ with the two qubits in some state $|\psi\rangle = |\psi_1\rangle|\psi_2\rangle$. (II) MRAP takes the qutrit to the state $(|B_1\rangle + |B_2\rangle)/\sqrt{2}$, (III) the controlled unitaries (U_1, U_2) are performed between each Bob site and the qubits, conditional on the presence of the qutrit at the appropriate site, and the system is transformed to $(|B_1\rangle|\psi'_1\rangle|\psi_2\rangle + |B_2\rangle|\psi_1\rangle|\psi'_2\rangle)/\sqrt{2}$. (IV) the transport is reversed and a projective measurement of the qutrit at $|A\rangle$ is performed. The results of this measurement projects the qubits into an eigenstate of the operator $U_1 U_2$, and depending on the measurement result, a phase flip at either $|B_1\rangle$ or $|B_2\rangle$ and further qutrit transport protocol can be used to return the qutrit to Alice.

For example, we could reexpress this interaction as either a CNOT or CZ gate where the control parameter is the presence or absence of the wavefunction of the qutrit at the nearest physical location. Such an interaction is physically motivated as, for example, we could consider a Coulomb interaction providing the entangling operation, and hence if the particle is not

present, no interaction will occur. The controlled gate is applied to Q_1 (Q_2) iff the qutrit state has non-zero amplitudes for $|B_1\rangle$ ($|B_2\rangle$) respectively. Alice transmits the particle to an equal superposition of the Bob sites and then the particle is coupled to the data qubits ($[Q_1, Q_2]$) through a CNOT (or CZ) operation. The protocol is schematically represented in Fig. 2. For total system state $|\Phi\rangle$ and a general two qubit state $|\psi\rangle_{Q_1, Q_2}$, these transformations are

$$U_{QTP}|\Phi\rangle = U_{QTP}|A\rangle \otimes |\psi\rangle = \frac{1}{\sqrt{2}}(|B_1\rangle + |B_2\rangle) \otimes |\psi\rangle$$

$$\xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|B_1\rangle X_{Q_1}|\psi\rangle + |B_2\rangle X_{Q_2}|\psi\rangle), \quad (6)$$

where $X_{Q_i}|\psi\rangle$ is a bit flip on qubit Q_i . The QTP is performed again, transforming the state to,

$$\frac{1}{2}|A\rangle(X_{Q_1}|\psi\rangle + X_{Q_2}|\psi\rangle) +$$

$$\frac{1}{2\sqrt{2}}(|B_1\rangle - |B_2\rangle) \otimes (X_{Q_1}|\psi\rangle - X_{Q_2}|\psi\rangle). \quad (7)$$

The system is then measured to determine if the qutrit has returned to the state $|A\rangle$. If it has, the information qubits are projected to $(X_{Q_1}|\psi\rangle + X_{Q_2}|\psi\rangle)$. If not, the information qubits are projected to $(X_{Q_1}|\psi\rangle - X_{Q_2}|\psi\rangle)$. Irrespective of the result after measurement, the transport qutrit is completely decoupled from the data qubits, therefore it can be discarded. However, if it needs to be reused and was not measured to the Alice state, a phase flip can be applied to B_2 taking, $(|B_1\rangle_1 - |B_2\rangle_1) \otimes (X_{Q_1}|\psi\rangle - X_{Q_2}|\psi\rangle) \rightarrow (|B_1\rangle_1 + |B_2\rangle_1) \otimes (X_{Q_1}|\psi\rangle - X_{Q_2}|\psi\rangle)$. After the phase flip the protocol can be deterministically reversed and the qutrit will return to the $|A\rangle$ state for reuse.

Inspection of the states generated by the protocol above, $(X_{Q_1}|\psi\rangle + X_{Q_2}|\psi\rangle)$ and $(X_{Q_1}|\psi\rangle - X_{Q_2}|\psi\rangle)$, reveals that if the qutrit is measured at Alice, the data qubits are projected to a $+1$ eigenstate of $X_{Q_1}X_{Q_2}$. If the qutrit is not measured at Alice then the data qubits are projected to a -1 eigenstate of $X_{Q_1}X_{Q_2}$. The same analysis can be performed using a CZ interaction between the qutrit and data qubits. In this case, the resultant state after measurement is projected to a ± 1 eigenstate of $Z_{Q_1}Z_{Q_2}$, dependant on whether the qutrit is measured in the Alice state.

As the protocol projects the data qubits into a ± 1 eigenstate of either XX or ZZ operators, loss of the qutrit during the protocol does not result in data loss: At most, the loss of the bus qutrit will simply induce a coherent X or Z error on one of the qubits as appropriate. A single qubit error is completely contained within the qubit space, and hence can be corrected via standard error correction protocols. Therefore, qutrit loss results in not knowing if the data qubits are in a $+1$ or -1 eigenstate of XX (ZZ). If the qutrit is lost, subsequent protocols will project to the data qubits to the same eigenstate. This leads to a repeat until success scheme [33, 34, 35] with no additional loss protocols required. For a practical device, therefore, it will be necessary to know the dephasing times

appropriate for the qutrit, however again, we note, that this scheme is still considerably more robust than a conventional flying qubit responding to an equally dephasing environment.

It is interesting to note, that the scheme as presented has much in common with the concept of mode entanglement [36, 37]. This topic has been the subject of much discussion recently due to the apparent contradiction between particle superposition and entanglement of modes that can be found in even simple beamsplitting experiments. A cursory glance of the qutrit transfer protocol above, shows that its action is analogous to the action of a beamsplitter on a single photon. One can therefore see that the reversals of the protocol provide a mechanism for the global measurement mentioned by Ashab *et al.* [38] in the context of massive particle mode-entanglement.

The QTP has identical properties to standard two qubit operator measurements [29]. However, instead of the usual ancilla qubit, which is always spatially localised, and interacts via (non-parallel) sequences of entangling gates, in our case, we employ a qutrit placed in a spatial superposition. In this superposition, each term acts as a control bit on *separate* spatially isolated data qubits. This couples $[Q_1, Q_2]$ via a qutrit bus which acts to mediate the entanglement. Alternatively, we can contrast the schemes as follows: the conventional circuit approach uses the computational state of the ancilla to tag certain data states, the QTP realises this tagging through the nonlocal degrees of freedom, essentially providing distributed entanglement in a natural fashion. The bus itself never carries any local data from either qubit, and after measurement becomes completely decoupled from the qubits: i.e. the bus is always information free. As the QTP can realise measurements of the operators XX and ZZ , the preparation of N qubit linear cluster states and universal computation can be achieved using these operator measurements and local unitaries on data qubits, as we will now show.

GENERATING CLUSTER AND STABILISER STATES

The concept of operator measurements is closely related to the Stabiliser formalism of Gottesman [39], commonly used in Quantum Error Correction (QEC) [40, 41, 42]. A state $|\Psi\rangle$ is *stabilised* by an operator U , if $U|\Psi\rangle = |\Psi\rangle$. For arbitrary operators, the stabiliser formalism is generally not useful, however there exists a certain class of states for which stabilisers provide a very elegant analytical tool. The Clifford group, \mathcal{C} , is a set of unitary operators, $\{O_j\} \in \mathcal{C}$, that under conjugation, map elements of the Pauli group, $P_j \in \mathcal{P}$, to themselves, $O_j^\dagger P_j O_j \in \mathcal{P}$, $\forall [O_j, P_j]$. A basis set for the Clifford group consists of CNOT, Hadamard, and S gate ($S \equiv \text{diag}\{1, i\}$). A general N qubit *stabilised* state, $|\Psi\rangle_N$, can be prepared by applying Clifford group operations to an initial $|00\dots 00\rangle_N$ state. Unlike arbitrary states, stabilised states can be described by N stabilisers, $\{G_j\}_N$, instead of the 2^N possible basis vectors. A stabilised state is therefore a si-

multaneous $+1$ eigenstate of each operator in $\{G_j\}_N$, which form an abelian subgroup of the N qubit Pauli group, i.e. $\{G_j\}_N \in \{I, X, Y, Z\}^{\otimes N}$. As stabilised states can be described by the N operators $\{G_j\}_N$, quantum circuits containing only Clifford group operations can be efficiently simulated classically [43]. Universality can be achieved by combining Clifford group operations with *any* single qubit gate that generates irrational rotations on the Bloch sphere [29].

Using the stabiliser formalism, highly entangled multi-qubit states, specifically GHZ and linear cluster states can be prepared when only small subset of two-qubit operator measurements and single qubit gates are available. The method requires the ability to initialise qubits in the $|0\rangle$ state, apply single qubit Hadamard, X and Z gates and the ability to perform two qubit XX and ZZ operator measurements. Combining the QTP introduced with the ability to do local operations directly on data qubits satisfies these conditions. GHZ preparation has already been considered [30], and the same methods can be extended to cluster states.

Raussendorf and Briegel [26, 27] demonstrated that any i -qubit cluster state, $|\text{CS}\rangle_i$, is defined by the eigenvalue equation, $K^{(a)}|\text{CS}\rangle_i = |\text{CS}\rangle_i$ where,

$$K^{(a)} = X_a \bigotimes_{b \in \text{ngbh}(a)} Z_b \quad \forall \quad a = 1 \dots i, \quad (8)$$

and $\text{ngbh}(a)$ represents qubits linked to site a in the cluster (neighbours), in arbitrary dimensions. Linear cluster states for 2 and 3 qubits are equivalent to Bell and three qubit GHZ states (up to local operations). For $N > 3$ the number of basis terms for cluster states grow quickly, hence it is better to express large cluster states via the eigenvalue equations. For example, a 4 qubit linear cluster state can be generated by the operators,

$$\begin{aligned} K^{(1)} &= XZII, & K^{(2)} &= ZXZI, \\ K^{(3)} &= IZXX, & K^{(4)} &= IIZX, \end{aligned} \quad (9)$$

where the \otimes signs are omitted for notational convenience. Since a four-qubit cluster state satisfies, $K^{(a)}|\text{CS}\rangle_4 = |\text{CS}\rangle_4$, the operators $K^{(a)}$ form a basis set of the stabiliser group for a 4 qubit linear cluster state. The stabiliser group can be used to specify the topology of a given cluster state, without having to write out the state directly.

Linear cluster state preparation using the QTP can be achieved by examining the stabiliser structure. The stabilisers for N qubits are generated by (neglecting identity operators), $K^{(1)} = X_1 Z_2$, $K^{(N)} = Z_{N-1} X_N$ and $K^{(j)} = Z_{j-1} X_j Z_{j+1}$, where $j = [2, 3, \dots, N-1]$. Preparing a state that satisfies this stabiliser structure using only XX and ZZ operator measurements, combined with direct single qubit operations requires linking the cluster together sequentially. To show the method explicitly, we detail the required steps needed to prepare a 4 qubit linear cluster state, after which adding links and expanding the cluster is straight forward. The analysis to follow assumes that we always measure the

+1 eigenstate of any given operator (i.e. the qutrit is measured at Alice), if the -1 eigenstate is obtained, simply apply local X and/or Z gates to correct. However, since X and Z gates are part of the Clifford group, all these corrections can be applied at the end of the state preparation.

Begin by initialising four qubits in the state $|\phi\rangle = |0\rangle_{Q_1}|0\rangle_{Q_2}|0\rangle_{Q_3}|0\rangle_{Q_4}$. The stabiliser group can be generated by the 4 operators, Z_j , $j = [1, 2, 3, 4]$. Measuring the operator $IXXI$, via the QTP, will project the state into a stabilised eigenstate of $IXXI$ and remove all existing stabilisers that anti-commute with $IXXI$. In this case, the stabilisers $IZII$ and $IIZI$ are removed, while $IZZII$ commutes with $IXXI$ and hence remains in the group. After measurement, the state of the computer will be stabilised by the basis operators, $K^{(1)} = IXXI$, $K^{(2)} = IZZI$, $K^{(3)} = IIIZ$ and $K^{(4)} = ZIII$. Qubits 2 and 3 are now in an entangled Bell state described by the basis stabilisers $IXXI$ and $IZZII$, while qubits 1 and 4 remain un-entangled. We now perform single qubit Hadamard rotation on qubit 1 which transforms X operators to Z and visa versa. This transforms the above basis stabilisers to, $K^{(1)} = IXXI$, $K^{(2)} = IZZI$, $K^{(3)} = IIIZ$ and $K^{(4)} = XIII$. Combining the stabilisers $[IXXI, XIII]$ and $[IZZII, IIIZ]$ shows that the qubits are also stabilised by the operators $[XXXI, IZZZ]$.

To produce the 4 qubit linear cluster state now requires $ZZII$ and $IIXX$ operator measurements. As these operator measurements act on independent qubits they can be performed in parallel given independent Alices. This will project the qubits into an eigenstate of these stabilisers and remove all previous non-commuting operators. From the above stabilisers, $K^{(1)} = IXXI$, $K^{(2)} = IZZI$, $K^{(3)} = IIIZ$ and $K^{(4)} = XIII$ anti-commute with either $ZZII$ or $IIXX$, while the stabilisers $XXXI$ and $IIIZ$ commute and hence remain in the set. The qubit register will now be in the stabilised state generated by the operators, $K^{(1)} = ZZII$, $K^{(2)} = XXXI$, $K^{(3)} = IZZZ$ and $K^{(4)} = IIXX$. If a Hadamard rotation is performed on qubits 1 and 3 the stabiliser group is rotated to,

$$\begin{aligned} K^{(1)} &= XZII, & K^{(2)} &= ZXZI, \\ K^{(3)} &= IZXZ, & K^{(4)} &= IIZX, \end{aligned} \quad (10)$$

which is the basis set of stabiliser operators describing a 4 qubit linear cluster state.

Extending this scheme to an N qubit linear cluster state is straightforward [Fig. 3]. Initially prepare N qubits (for simplicity assume N even) in the state $|\phi\rangle_N = |00\dots00\rangle_N$. The stabiliser group for this state is generated by Z_j , $j = [1, 2, \dots, N]$. Hadamard rotations are performed on all odd numbered qubits except for the two at the centre of the chain and the following operators are measured,

$$\begin{aligned} \text{Step 1} & \quad X_{N/2}X_{N/2+1} \\ \text{Step 2} & \quad Z_{N/2-1}Z_{N/2} \quad \text{and} \quad X_{N/2+1}X_{N/2+2} \\ \text{Step 3} & \quad X_{N/2-2}X_{N/2-1} \quad \text{and} \quad Z_{N/2+2}Z_{N/2+3} \\ \text{Step } N/2 & \quad Z_1Z_2 \quad \text{and} \quad X_{N-1}X_N. \end{aligned} \quad (11)$$

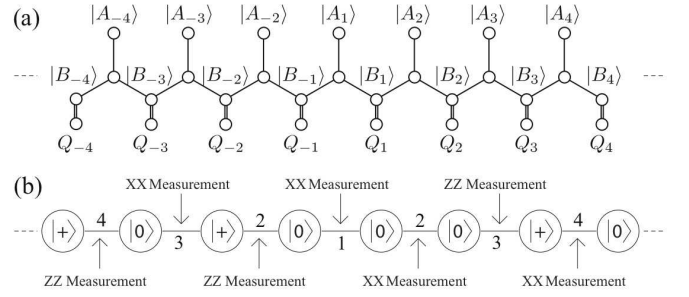


FIG. 3: (a) Schematic of multi-Alice multi-Bob structure for generating stabilised states, the Alices are labelled so that at time step 1, Alice₁ is used, and at all subsequent time steps i , Alice _{i} and Alice _{$-i$} are used, with coupling to Bobs as indicated. (b) Required two qubit operator measurements to prepare an N qubit linear cluster state. Each of the qubits illustrated (circles) is initialised in the $|0\rangle$ state, Hadamard rotations performed on specific qubits (rotating to the $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ state) and XX or ZZ operator measurements made. Each link is labelled by the respective time step in which it can be performed, assuming that independent operator measurements can be done in parallel by defining an isolated ($|A\rangle, |B_1\rangle, |B_2\rangle$) channel in the bus for each operator measurement.

Hadamard gates are again applied to all odd numbered qubits, after which the generators of the stabiliser set are identical to a linear cluster state. Although additional links in the cluster are created sequentially, the first link is made from the centre of the chain and subsequent links formed from this point, increasing the number of possible operations that can be done in parallel. Using this method, an N qubit linear cluster state can be prepared using $N/2$ time steps (for N even) or $N/2 + 1$ time steps (for N odd).

The preparation of linear cluster states via the QTP is useful in the preparation of multi-qubit entangled systems, however Nielsen [44] has shown that linear cluster states are insufficient for universal quantum computation. The original proposal of Raussendorf and Briegel suggested a 2-D tiled cluster state be used, however the QTP combined with single qubit gates is insufficient to create such a state directly. Universality can be achieved if we employ the results of Aliferis and Leung [22] and their work into TQC. The QTP already assumes that single qubit gates can be performed directly on data qubits, hence the ability to simulate *any* entangling gate between two data qubits is sufficient for universality [45]. We demonstrate explicitly how a CZ gate can be simulated using the QTP and direct single qubit operations.

Consider an arbitrary two qubit state $|\psi\rangle_{12} = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$ and a third data qubit prepared in a $|+\rangle_3 = (|0\rangle_3 + |1\rangle_3)/\sqrt{2}$ state that acts as an ancilla. Using the QTP a Z_1Z_3 operator measurement is performed on the control and ancilla qubit (again we assume that the qutrit returns to Alice and +1 eigenstates are projected, if not the classical measurement record can be used to correct the state using X and/or Z gates). This operation takes the combined qubit/ancilla state to $|\psi'\rangle = |\psi\rangle \otimes |+\rangle \rightarrow \alpha|000\rangle + \beta|010\rangle - \gamma|101\rangle - \delta|111\rangle$. A Hadamard gate is applied to the target

qubit taking the combined qubit/ancilla state to $|\psi'\rangle = \alpha|0+0\rangle + \beta|0-0\rangle - \gamma|1+1\rangle - \delta|1-1\rangle$. An X_2X_3 operator measurement is now performed on the ancilla and target qubits taking the state to $|\psi'\rangle = \alpha|0++\rangle - \beta|0--\rangle - \gamma|1++\rangle - \delta|1--\rangle$. Performing a Hadamard rotation on the target qubit leads to

$$|\psi'\rangle = (\alpha|00\rangle - \beta|01\rangle - \gamma|10\rangle - \delta|11\rangle)_{12} \otimes |0\rangle_3 + (\alpha|00\rangle + \beta|01\rangle - \gamma|10\rangle + \delta|11\rangle)_{12} \otimes |1\rangle_3. \quad (12)$$

The ancilla qubit is now measured in the computational basis. If it is measured in the $|0\rangle$ state, local phase gates are applied to both the control and target qubit. If the ancilla is measured in the $|1\rangle$ state, a local phase gate is applied to the control qubit. After these corrections the state has been transformed from $|\psi\rangle$ to $\text{CZ}|\psi\rangle$. Therefore, using specific operator measurements and local gates, a CZ gate can be effectively simulated across two qubits by introducing a third ancilla.

Since a CZ gate can be directly implemented in this scheme, and we have assumed that single qubit operations can be implemented directly on data qubits, universal computation is possible using the QTP. The simulation of CZ gates also allows for the preparation of arbitrary cluster states (if desired) using the standard method of linking un-entangled $|+\rangle$ states with CZ gates.

CONCLUSIONS

We have presented an information-free quantum bus, based on qutrits that acts to mediate entanglement between data qubits pairwise. To clarify this protocol, we have explicitly shown an implementation that uses spatial adiabatic passage with a spatially defined qutrit. Our protocol allows for deterministic XX and ZZ operator measurements to be performed on separate data qubits. We have demonstrated how this restricted set of operator measurements, combined with the ability to do single qubit operations directly on data qubits, allows for the preparation of N qubit linear cluster states and simulation of controlled phase (CZ) gates between two data qubits. This approach to direct synthesis of operator measurements may have significant application to improving the efficiency of quantum operations, and constitutes a different approach to the generation of remote entanglement from flying qubit methods.

The authors thank J.Cole, A.Fowler, and K. Maruyama for helpful discussions. This work was supported by the Australian Research Council, US National Security Agency (NSA), Advanced Research and Development Activity (ARDA) and Army Research Office (ARO) under contract W911NF-04-1-0290.

- [2] T. Schaetz *et al.*, Phys. Rev. Lett. **93**, 040505 (2004).
- [3] S.F. Huelga, C.Macchiavello, T.Pellizzari, A.K. Ekert, Phys. Rev. Lett. **79**, 3865 (1997).
- [4] J.I. Cirac and P. Zoller, Nature (London), **404**, 579 (2000).
- [5] D. Kielpinski, C. Monroe, D.J. Wineland, Nature (London), **417**, 709 (2002).
- [6] E. Knill, R. Laflamme and G. Milburn, Nature (London), **409**, 46 (2001).
- [7] D. Loss and D.P. DiVincenzo, Phys. Rev. A. **57**, 120 (1998).
- [8] B.E. Kane, Nature (London), **393**, 133 (1998).
- [9] R. Vrijen *et al.*, Phys. Rev. A. **62**, 012306 (2000).
- [10] T. D. Ladd, J. R. Goldman, F. Yamaguchi, Y. Yamamoto, E. Abe and K. M. Itoh, Phys. Rev. Lett. **89**, 017901 (2002).
- [11] M. Friesen *et al.*, Phys. Rev. B. **67**, 121301 (2003).
- [12] R. de Sousa, J. D. Delgado and S. Das Sarma, Phys. Rev. A **70**, 052304 (2004).
- [13] C. D. Hill, L. C. L. Hollenberg, A. G. Fowler, C. J. Wellard, A. D. Greentree, and H.-S. Goan, Phys. Rev. B **72**, 045350 (2005).
- [14] T. Szkopek *et al.*, IEEE Trans. Nano., **5**, 42 (2006).
- [15] D.P. DiVincenzo, Fortschr. Phys. **48**, 771 (2000).
- [16] A.J. Skinner, M.E. Davenport and B.E. Kane, Phys. Rev. Lett. **90**, 087901 (2003).
- [17] A.D. Greentree, J.H. Cole, A.R. Hamilton, L.C.L. Hollenberg, Phys. Rev. B. **70**, 235317 (2004).
- [18] T.P. Spiller, K. Nemoto, S. L. Braunstein, W. J. Munro, P. van Loock, G. J. Milburn, New J. Phys. **8**, 30 (2006).
- [19] L.C.L. Hollenberg, A.D. Greentree, A.G. Fowler, C.J. Wellard, Phys. Rev. B **74** 045311 (2006).
- [20] S. Bose, Phys. Rev. Lett. **91**, 207901 (2003).
- [21] M.H. Oskin, F.T. Chong, I.L. Chuang, IEEE Computer **35**, 79 (2002).
- [22] P. Aliferis, D.W. Leung, Phys. Rev. A. **70**, 062314 (2004).
- [23] D. Gottesman, I.L. Chuang, Nature (London), **402**, 390 (1999).
- [24] D.W. Leung, Int. Journal of Quantum Information, **2**, 33 (2004).
- [25] D.W. Leung, quant-ph/0111122, (2001).
- [26] R. Raussendorf, H. J. Briegel, Phys. Rev. Lett. **86**, 5188 (2001).
- [27] R. Raussendorf, D.E. Browne, H.J. Briegel, Phys. Rev. A. **68**, 022312 (2002).
- [28] M. Hein, J. Eisert, H.J. Briegel, Phys. Rev. A. **69**, 062311 (2004).
- [29] M. A. Nielsen and I.L. Chuang, Quantum computation and quantum information. Cambridge University Press, (2000).
- [30] A.D. Greentree, S.J. Devitt, and L.C.L. Hollenberg, Phys. Rev. A **73**, 032319 (2006).
- [31] R. G. Unanyan, B. W. Shore, and K. Bergmann, Phys. Rev. A **59**, 2910 (1999).
- [32] M. D. Lukin and P. R. Hemmer, Phys. Rev. Lett. **84**, 2818 (2000).
- [33] Y. L. Lim, A. Beige, and L. C. Kwek Phys. Rev. Lett. **95**, 030505 (2005).
- [34] S. D. Barrett and P. Kok, Phys. Rev. A **71** 060310(R) (2005).
- [35] Y. L. Lim, S. D. Barrett, A. Beige, P. Kok and L. C. Kwek, Phys. Rev. A **73** 012304 (2006).
- [36] S. M. Tan, D. F. Walls, and M. J. Collett, Phys. Rev. Lett. **66**, 252 (1991).
- [37] L. Hardy, Phys. Rev. Lett. **73**, 2279 (1994).
- [38] S. Ashhab, K. Maruyama, and F. Nori, Phys. Rev. A **75**, 022108 (2007).
- [39] D. Gottesman, Ph.D Thesis, Caltech quant-ph/9705052, (1997).
- [40] A. M. Steane, Phys. Rev. Lett. **77**, 793 (1996).
- [41] P.W. Shor, Phys. Rev. A. **52**, R2493 (1995).
- [42] A.R. Calderbank, P.W. Shor, Phys. Rev. A. **54**, 1098 (1996).
- [43] D. Gottesman, Phys. Rev. A **57**, 127 (1998).

- [44] M.A. Nielsen, Reports on Mathematical Physics **57**, 147 (2006).
[45] M.J. Bremner, *et al.*, Phys. Rev. Lett. **89**, 247902 (2002).